A Simple Way to Model Curved Metal Boundaries in FDTD Algorithm Avoiding Staircase Approximation

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Abstract—**The conventional FDTD algorithm in Cartesian coordinates uses staircase approximation to treat curvilinear surfaces. This approximation causes loss of accuracy often unacceptable. An extremely simple and more accurate polygonal approximation of curved surfaces is proposed in this paper. The method improves significantly the accuracy of the original FDTD algorithm, without increasing its complexity.**

I. Introduction

Because of its high flexibility and accuracy, the FDTD method is increasingly popular in analyzing microwave and millimeterwave structures. Unfortunately, in its simplest formulation, that employs an orthogonal mesh [1], [2], the method does not provide a good accuracy when curved surfaces are present. This is mainly due to the staircase approximation of curvilinear surfaces. Several methods have been proposed in the literature [3]–[7] to conform the grid to arbitrarily shaped structures, but they require additional computational cost and great implementation effort. In this paper we propose a very simple and accurate procedure to approximate a metal curved surface within a Cartesian mesh. The method allows the conventional staircase approximation to be replaced with a polygonal approximation, thus highly improving the accuracy level.

II. Modeling SlantedWalls

Consider a rectangular cell, located at a metallic boundary, that crosses the cell along its diagonal (Fig. 1). The proper FDTD formulation for the fields relevant to the cell is obtained from the integral form of Maxwell's equations

$$\oint \vec{H}\cdot d\vec{l} = \int\!\!\int_s \left(\varepsilon\frac{d\vec{E}}{dt} + \sigma\vec{E}\right)\cdot d\vec{s} \tag{1}$$

$$\oint \vec{E}\cdot d\vec{l} = \int\!\!\int_s \mu\frac{d\vec{H}}{dt}\cdot d\vec{s}. \tag{2}$$

Applying (2) to the cell of Fig. 1, one obtains

$$\begin{aligned}&E_x^n\left(i+\frac{1}{2}, j+1, k\right)\Delta x + E_y^n\left(i, j+\frac{1}{2}, k\right)\Delta y\\ &\quad + E_{\tan g}^n\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\sqrt{\Delta x^2+\Delta y^2}\\ &= \left(H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2}, J+\frac{1}{2}, k\right) - H_z^{n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\right)\\ &\quad\times\frac{\Delta x\Delta y}{2\Delta t}\end{aligned} \tag{3}$$

where, according to the Yee's formulation [1], $\Delta x, \Delta y, \Delta z$ are the space steps, Δt is the time step, the superscript indicates the time index and i, j, and k are the space coordinate indexes. $E_{\tan g}$ is the E-field component along the cell diagonal and is therefore zero on the metallic wall. From (3) H_z can be easily calculated as follows:

$$\begin{aligned}&H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\\ &= H_z^{n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\\ &\quad + \frac{2\Delta t}{\Delta x\Delta y}\left(E_x^n\left(i+\frac{1}{2}, j+1, k\right)\Delta x\right.\\ &\quad\left. + E_y^n\left(i, j+\frac{1}{2}, k\right)\Delta y\right).\end{aligned} \tag{4}$$

Observe that the conventional stairstep evaluation of the same H component gives

$$\begin{aligned}&H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\\ &= H_z^{n-\frac{1}{2}}\left(i+\frac{1}{2}, j+\frac{1}{2}, k\right)\\ &\quad + \frac{\Delta t}{\Delta x\Delta y}\left(E_x^n\left(i+\frac{1}{2}, j+1, k\right)\Delta x\right.\\ &\quad\left. + E_y^n\left(i, j+\frac{1}{2}, k\right)\Delta y\right).\end{aligned} \tag{5}$$

The modified formula (4) for the triangular cell differs from the conventional one (5) only for the presence of the 2 factor in the right hand side. This simply corresponds to the cell having half the surface and can also be seen as a particular case of the general formulation given in [7]. This technique can be used to conform the mesh to any slanted plane wall by choosing the proper aspect ratio of the cells.

III. Modeling Curved Surfaces

In order to extend the technique described to approximating curved surfaces, a graded mesh implementation of the original FDTD algorithm can be adopted [8]. By a proper choice of the mesh grading it is possible to locate the boundary nodes of the mesh exactly on the curved surface, in such a way as to approximate the arc laying on a cell diagonal with the diagonal itself (see Fig. 5).

IV. Results

In order to test the accuracy of the proposed formula, a simple rectangular cavity with dimension 10∗ 10∗ 18 mm has first been simulated using three different discretizations:

Manuscript received February 22, 1995.

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IEEE Log Number 9412823.

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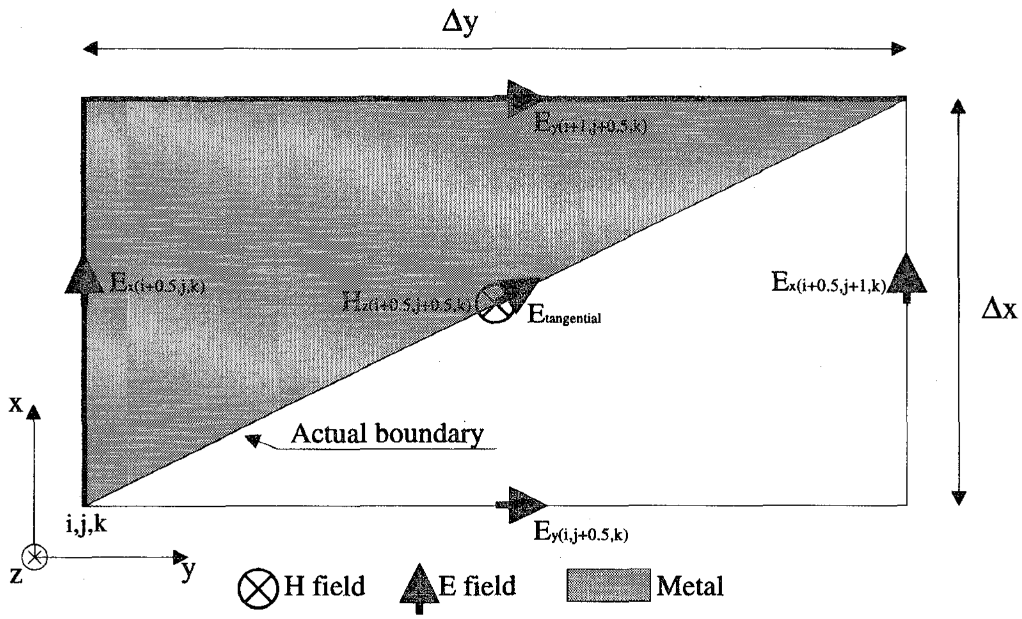

Fig. 1. Enlargement of a cell close to the slanted metallic surface.

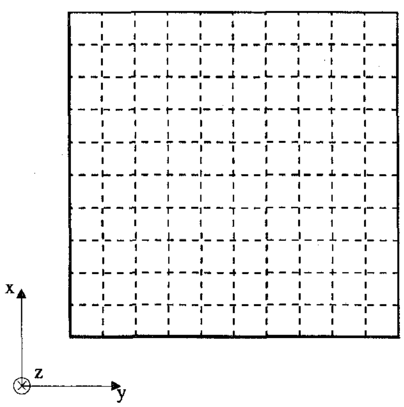

Fig. 2. Cross-section of a rectangular resonant cavity perfectly fitting to the orthogonal mesh.

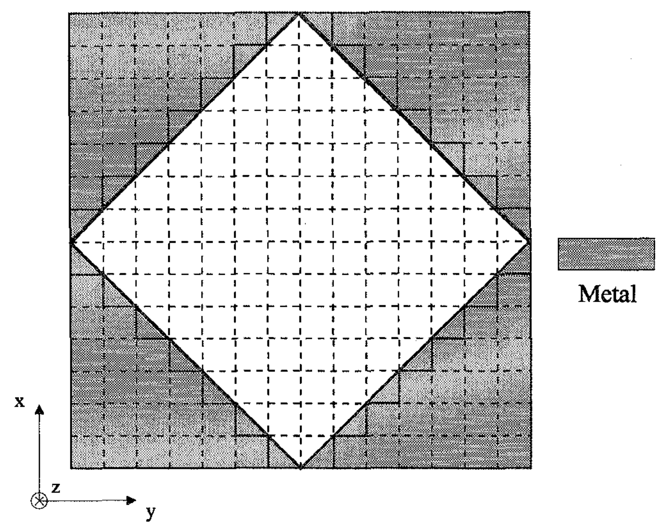

Fig. 3. Cross-section of the same cavity of Fig. 2, 45° slanted and treated with the proposed formula.

TABLE I
COMPARISON AMONG THEORETICAL AND NUMERICAL RESULTS (UNIT = GHz)

METHOD	TE_{011}	TE_{101}	TM_{110}	TE_{012}	TE_{102}	TM_{111}	TE_{111}
Theory	17.147	17.147	21.198	22.407	22.407	22.775	22.775
Orthogonal fitting	17.1	17.1	21.2	22.4	22.4	22.8	22.8
Staircase	16.7	16.7	22.6	21.5	21.5	24.1	22.1
New formula	17.1	17.1	21.1	22.4	22.4	22.7	22.7

1) a conventional rectangular mesh perfectly fitting the geometry of the cavity (Fig. 2)
2) a slanted cavity with new evaluation of boundary H fields (Fig. 3)
3) a slanted cavity with the best staircase approximation.

The cavity has been discretized in all cases with approximately the same number of cells.

Table I shows the theoretical and calculated resonant frequencies for the first five modes of the cavity for the three different discretizations.

The staircase approximation is seen to deteriorate the accuracy of the method, the error being of the order of 2 ÷ 6%. With the same numerical effort, the introduction of the triangular cells at the metal boundary restores the same accuracy of the orthogonal mesh.

The proposed formula has then been combined with a proper mesh grading. The technique has been applied to the analysis of the cylindrical cavity measured by Fontecha *et al.* [9] and analyzed by Niu *et al.* [10] (Fig. 4).

Fig. 5 shows how the mesh matches both the cylindrical surface and the feeding rectangular waveguides. Experimental data and FDTD with staircase approximation from [10] are

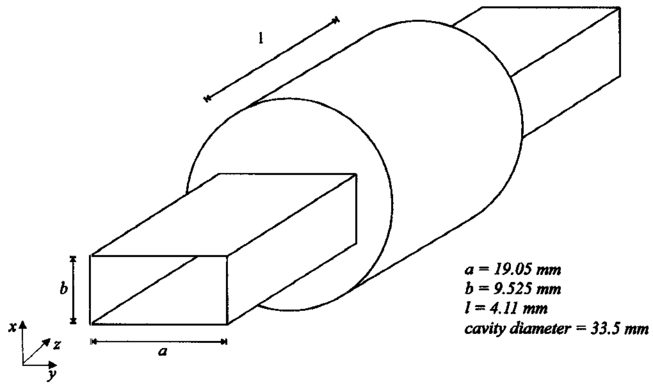

Fig. 4. Sketch of the simulated cylindrical cavity.

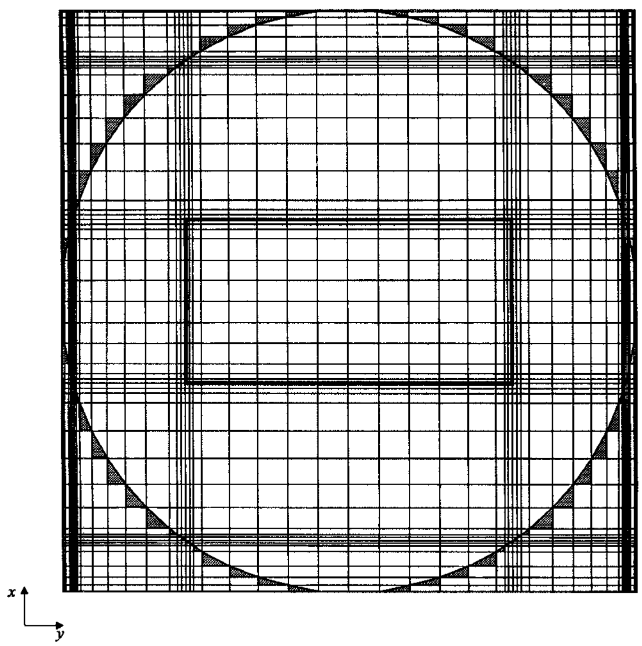

Fig. 5. Transverse section of the mesh.

compared with the proposed FDTD new implementation and with a mode matching result. The cylindrical cavity is discretized here by 51∗51∗13 = 33813 cells instead of 72∗ 72∗ 8 = 41472 as in [10]; moreover, the feeding lines are terminated by unimodal absorbing boundary conditions [11], [12]. This termination allows for a great mesh saving: only 16∗16∗34 cells have been required to model the feeding lines instead of 40∗20∗100 as in [10]. Thus, a global mesh saving greater than 4 times is attained. It should be observed however that a time step of 0.614 ps instead of 0.925 ps has been used in order to comply with the stability criterion. In spite of the shorter time step, the computational effort for the analysis of the structure in Fig. 4 has been reduced: a computational saving of about 3 times has been achieved with a resulting error one order of magnitude smaller (0.21% instead of 2.15%)

V. Conclusion

A new very simple method for treating curved metal boundaries by FDTD has been proposed. An improvement of the efficiency of more then one order of magnitude has been demonstrated. The method can easily be extended to curved dielectric interfaces.

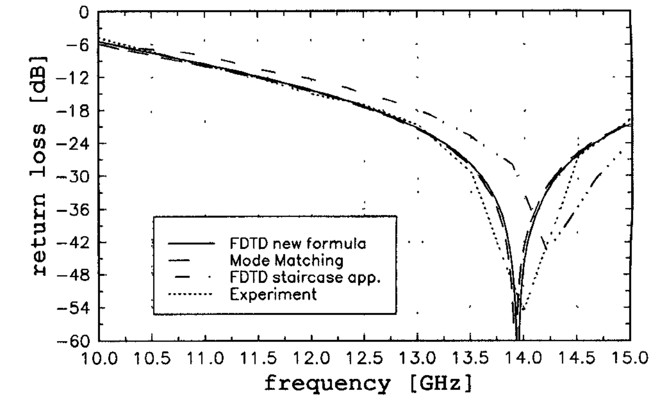

Fig. 6. Comparison of final results (Experiments and FDTDstaircase data are published in [8]).

Acknowledgment

The authors gratefully acknowledge Dr. S. Moretti for providing mode matching data.

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